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TECHNICAL NOTE 2125

PRANDTL-MEYER FLOW FOR A DIATOMIC GAS OF
VARIABLE SPECIFIC HEAT

By Robert N. Noyes

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PRANDTL-MEYER FLOW FOR A DIATOMIC GAS OF
VARIABLE SPECIFIC HEAT

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SUMMARY

Tables and charts, which give the results of an analysis that accounts for variation in specific heats of a nonviscous compressible fluid (diatomic gas) during the Prandtl-Meyer flow process (commonly called the flow around a corner), are presented. Comparison is made to the constant-specific-heat solution with the ratio of specific heats $\gamma = 1.4$ and with this ratio corresponding to the total fluid temperature. The comparison showed that variation in specific heats appreciably affected the magnitude of some of the parameters pertinent to this flow but that a close approximation to the variable-specific heat solution could be obtained by the use of a constant value of the ratio of specific heats corresponding to the total fluid temperature.

INTRODUCTION

In the supersonic flow of a fluid from a high- to a relatively low-pressure region, the resultant expansion is accomplished through the medium of expansion waves, which simultaneously turn and diverge the fluid streamlines. Supersonic flow of a fluid around a sharp convex corner of a wall represents one such expansion process. Understanding of this expansion process is of fundamental importance in some supersonic-flow problems inasmuch as the solution for this particular process can be used to describe other expansion processes, such as expansion along the curved boundary of a two-dimensional supersonic wing or supersonic nozzle, or expansion in a free-jet stream such as would exist in underexpanding supersonic nozzles or in the clearance space between turbine nozzles and turbine blades.

The solution of the corner flow of a nonviscous compressible fluid given by Prandtl and Meyer is based on the assumption that the ratio of specific heats remains constant during the expansion. When high fluid temperatures and high expansion pressure ratios are simultaneously involved, however, the variation of specific heats during the process may become appreciable.

In a study made at the NACA Lewis laboratory, the Prandtl-Meyer solution has been extended to the case of a diatomic gas having variable specific heats. The effect of specific-heat lag on the flow process is assumed negligible. The results are presented in convenient tabular and chart form. The flow variables presented are streamline angle, ray angle, Mach angle, local Mach number, pressure ratio, temperature ratio, ratio of local velocity to velocity at a Mach number of unity, and the ratio of the density-velocity product to its value at a Mach number of unity. The range of variables investigated corresponds to streamline angles from 0° to 50° .

SYMBOLS

The following symbols are used in this report:

| | |
|------------------|--|
| C_p | specific heat at constant pressure |
| E_i | internal energy per unit mass of gas |
| E_v | vibrational energy per unit mass of gas |
| H | total enthalpy per unit mass of gas |
| k | constant resulting from method of removing singularity |
| M | Mach number |
| M_o | initial Mach number |
| P | total pressure |
| p | static pressure |
| R | gas constant |
| ΔS° | change in dilute-phase entropy |
| Δs | change in total entropy |
| T | absolute total temperature |
| t | absolute static temperature |
| u | velocity in direction of radius vector |

- V vector velocity having components u and v
 v velocity in direction perpendicular to radius vector
 α streamline angle ($\alpha = 0$ for $M = 1$), degrees
 α_0 streamline angle resulting from hypothetical expansion from $M = 1$ to initial Mach number, degrees
 β Mach angle, degrees
 γ ratio of specific heats
 θ characteristic temperature of molecular vibration
 ρ density of gas
 τ θ/t
 ϕ ray angle ($\phi = 0$ for $M = 1$), degrees
 ϕ_0 ray angle resulting from hypothetical expansion from $M = 1$ to the initial Mach number, degrees

Superscript:

- * conditions in gas when $M = 1$

METHOD OF ANALYSIS

A brief description of the method of analysis of the flow of a nonviscous compressible fluid of variable specific heat around a corner is presented; the details are given in the appendix.

The trace of a streamline in flowing around a corner boundary is schematically shown in figure 1. A gas initially flowing along a wall at a Mach number $M > 1$ approaches the corner at which an isentropic expansion occurs. The expansion is accomplished through the medium of expansion waves, which emanate from the corner. As a result of the expansion, a streamline will be turned through an angle $\alpha - \alpha_0$ at a Mach line OM , where α_0 is the streamline angle resulting from a hypothetical expansion of the uniform stream from a Mach number of unity to the Mach number M_0 and α is the streamline angle for an expansion from a Mach number of unity to any

Mach number. The fluid continues to expand until the static pressure in the stream is equal to the ambient pressure or until the flow is parallel to the wall. Associated with the deflection angle $\alpha - \alpha_0$ are the ray angle $\phi - \phi_0$ and the Mach angle β .

The solution of this flow was obtained by simultaneously solving the equations of motion, (equations (A1) and (A2) in appendix), the continuity equation (equation (A3) in appendix), and the energy equation. At sufficiently low temperatures, the energy equation needs to include only the internal energy due to rotation and translation of the molecules, and the kinetic energy and flow energy of the gas stream. As the temperature of a diatomic gas is increased, the energy of vibration of the molecules becomes significant and thus should be included in the energy equation. It is this energy contribution that is reflected as a variation of the specific heats.

By assuming the rotational energy to be that of a rigid rotator, the vibrational energy of a diatomic gas in which the molecules oscillate harmonically was obtained from the kinetic theory of gases (reference 1) by

$$E_v = Rt \left(\frac{\tau}{e^{\tau} - 1} \right) \quad (1)$$

The internal energy of the gas, including the vibration energy in addition to the translational and rotational energy, was then given by

$$E_i = Rt \left(\frac{5}{2} + \frac{\tau}{e^{\tau} - 1} \right) \quad (2)$$

and the energy equation for a conservative system thus became

$$H = \frac{V^2}{2} + \left(\frac{7}{2} + \frac{\tau}{e^{\tau} - 1} \right) Rt = \text{constant} \quad (3)$$

Equation (3), together with the equations of motion and the continuity equation were simultaneously solved to obtain a single differential equation relating the ray angle to the static gas temperature. This equation was integrated by numerical means and from the results other parameters pertinent to the corner flow solution were obtained.

RESULTS OF ANALYSIS

The results of the analysis of the flow of a nonviscous compressible diatomic gas of variable specific heat around a corner for two different ratios of total fluid temperature to characteristic temperature of molecular vibration T/θ are shown in table I. Shown in the table as functions of the streamline angle α (fig. 1) are the ray angle ϕ , the Mach angle β , the ratio of static pressure to total pressure p/P , the local Mach number M , the ratio of fluid velocity to its value at a Mach number of unity V/V^* , the ratio of static temperature to total temperature t/T , and the mass-flow ratio $\rho V/\rho^* V^*$. The ratio $\rho V/\rho^* V^*$ is the ratio of the product of the density and fluid velocity to the value of this product at a Mach number of unity.

In figure 2 the calculated results, given in table I, are compared with results obtained by the conventional constant-specific-heat solution where $\gamma = 1.4$, as obtained from reference 2. These data are tabulated in table II. The error that results from use of the conventional constant-specific-heat computations (fig. 2) can be appreciable at high temperatures. The value $\gamma = 1.4$, however, is known to be incorrect at these temperatures. Use of a constant value of γ corresponding to the total temperature of the fluid considerably reduces the error. This result is shown in figure 3, where the percentage error resulting from use of $\gamma = 1.303$ (which corresponds to $T/\theta = 0.6$ (see equation (A26) of appendix)) is plotted against streamline angle α for each of the flow parameters considered. (Percentage error, divided by 100, is the value of a parameter computed for $\gamma = 1.303$ minus the value of the parameter computed for a variable ratio of specific heat all divided by the value of the parameter computed for the γ variable.) For a streamline angle of 40° , the pressure ratio and the temperature ratio are in error by approximately 3 percent; the remaining flow parameters are in error by 0.5 percent or less (fig. 3). Smaller errors in all parameters would be involved for values of T/θ less than the value of 0.6, which was used in the comparison of figure 3.

In order to use the results of the variable-specific-heat analysis in a practical problem, the characteristic temperature of gas vibration must be known. This temperature is a constant only for a single diatomic gas. For air and lean exhaust-gas mixtures, however, the variation of θ with exhaust-gas temperature is so small that an average value of θ may be chosen that will closely fit experimental specific-heat data in the region of interest. Values of θ (obtained by substituting experimental specific-heat

data in equation (A26) in appendix) recommended for use in the tables and charts for air, for products of combustion with 400 percent of theoretical air, and for products of combustion with 200 percent of theoretical air are 5450° R, 4500° R and 3900° R, respectively.

SUMMARY OF RESULTS

The results of calculation of the flow of a nonviscous compressible diatomic gas around a corner showed that the effect of variation in specific heat on the magnitude of parameters pertinent to this flow is appreciable at high temperatures and high expansion pressure ratios. An analysis showed that in the absence of the variable-specific-heat tabulations presented very close approximations to the true value of the parameters can be calculated when a constant specific-heat value corresponding to the total fluid temperature is assumed.

Lewis Flight Propulsion Laboratory,
National Advisory Committee for Aeronautics,
Cleveland, Ohio, November 15, 1949.

APPENDIX - DETAILS OF ANALYSIS

For the steady nonviscous flow of a compressible fluid around a corner, all derivatives with respect to the radius vector are zero. The equations of motion and the continuity equation are thus (fig. 1):

$$\frac{du}{d\phi} - v = 0 \quad (A1)$$

$$v \frac{dv}{d\phi} + uv = - \frac{1}{\rho} \frac{dp}{d\phi} \quad (A2)$$

$$\rho u + \frac{d}{d\phi} (\rho v) = 0 \quad (A3)$$

By assuming the rotational energy of a diatomic gas to be that for a rigid rotator and the vibrational energy to be that for a harmonic oscillator, the vibrational energy is

$$E_v = Rt \left(\frac{\tau}{e^{\tau} - 1} \right) \quad (1)$$

With this relation, the internal energy is then given by

$$E_i = Rt \left(\frac{5}{2} + \frac{\tau}{e^{\tau} - 1} \right) \quad (2)$$

and the total enthalpy by

$$H = \frac{u^2 + v^2}{2} + E_i + Rt = \text{constant} \quad (A4)$$

Substitution of equation (A1), that is, $v = du/d\phi$, into equations (A2), (A3), and (A4) results in the following equations:

$$\frac{du}{d\phi} \left(\frac{d^2u}{d\phi^2} \right) + u \frac{du}{d\phi} = - \frac{1}{\rho} \frac{dp}{d\phi} \quad (A5)$$

$$\rho u + \rho \frac{d^2u}{d\phi^2} + \frac{du}{d\phi} \left(\frac{d\rho}{d\phi} \right) = 0 \quad (A6)$$

$$u^2 + \left(\frac{du}{d\phi}\right)^2 + 2(E_i + Rt) = 2H \quad (A7)$$

Because for a perfect gas $p = \rho R t$, equation (A5) may be expanded in the form

$$\frac{du}{d\phi} \left(\frac{d^2 u}{d\phi^2} \right) + u \frac{du}{d\phi} = - Rt \left(\frac{d \log_e \rho}{d\phi} + \frac{d \log_e t}{d\phi} \right) \quad (A8)$$

From equation (A6)

$$\frac{d \log_e \rho}{d\phi} = - \left(u + \frac{d^2 u}{d\phi^2} \right) \frac{\frac{du}{d\phi}}{\left(\frac{du}{d\phi} \right)^2} \quad (A9)$$

Hence equation (A8) becomes

$$\left[\left(\frac{du}{d\phi} \right)^2 - Rt \right] \left(u + \frac{d^2 u}{d\phi^2} \right) + R \left(\frac{du}{d\phi} \right) \left(\frac{dt}{d\phi} \right) = 0 \quad (A10)$$

From equation (A7) after differentiating with respect to ϕ

$$u + \frac{d^2 u}{d\phi^2} = - \left[\frac{\frac{dE_i}{dt} \left(\frac{dt}{d\phi} \right) + R \frac{dt}{d\phi}}{\frac{du}{d\phi}} \right] \quad (A11)$$

From equations (A10) and (A11),

$$\left[\left(\frac{du}{d\phi} \right)^2 - Rt \right] \left(\frac{dE_i}{dt} + R \right) - R \left(\frac{du}{d\phi} \right)^2 = 0 \quad (A12)$$

and thus

$$\frac{du}{d\phi} = \sqrt{\frac{Rt}{dE_i/dt} \left(\frac{dE_i}{dt} + R \right)} \quad (A13)$$

By substituting equation (A13) in equation (A7)

$$u = \sqrt{2(H - E_1 - Rt) - \frac{Rt}{dE_1/dt} \left(\frac{dE_1}{dt} + R \right)} \quad (A14)$$

The variable u and its derivatives may now be substituted into equation (A10) so as to give a single equation involving t and ϕ . This equation is of the form

$$-\tau^2 \left(\frac{d\phi}{d\tau} \right) = \frac{A-B}{D} \quad (A15)$$

where

$$A = \frac{e^\tau}{4} \left(\frac{2\tau}{e^\tau - 1} \right)^2 \left(\tau - 2 + \frac{2\tau}{e^\tau - 1} \right) \quad (A16)$$

$$B = 2 \left[\frac{7}{2} + \frac{\tau^2 e^\tau}{(e^\tau - 1)^2} \right] \left[\frac{6}{2} + \frac{\tau^2 e^\tau}{(e^\tau - 1)^2} \right] \left[\frac{5}{2} + \frac{\tau^2 e^\tau}{(e^\tau - 1)^2} \right] \quad (A17)$$

$$D = 2 \left[\frac{5}{2} + \frac{\tau^2 e^\tau}{(e^\tau - 1)^2} \right] \sqrt{\frac{2}{\tau} \left(\frac{H}{R\theta} - \frac{7}{2\tau} - \frac{1}{e^\tau - 1} \right) \left[\frac{5}{2} + \frac{\tau^2 e^\tau}{(e^\tau - 1)^2} \right] \left[\frac{7}{2} + \frac{\tau^2 e^\tau}{(e^\tau - 1)^2} \right] - \frac{1}{\tau^2} \left[\frac{7}{2} + \frac{\tau^2 e^\tau}{(e^\tau - 1)^2} \right]^2} \quad (A18)$$

When $\tau = \tau^*$, which for a given θ corresponds to the value of the static temperature in the gas when the local Mach number is unity, the denominator term D is zero. This singularity was investigated and found to be a pole of order minus one half. It was removed by assuming that

$$\frac{d\phi}{d\tau} = \frac{k}{\sqrt{\tau - \tau^*}} + X(\tau) = \frac{1}{\tau^2} \left(\frac{B}{D} - \frac{A}{D} \right) \quad (A19)$$

where $X(\tau)$ is everywhere finite and has finite derivatives and $X(\tau^*) = 0$. The constant k was computed by evaluating, for each value of $\frac{H}{R\theta}$, the expression

$$k = \left(\frac{B-A}{\tau^2} \right) \lim_{\tau \rightarrow \tau^*} \frac{\sqrt{\tau - \tau^*}}{D} \quad (A20)$$

In the integration of equation (A19), seven point formulas obtained from reference 3 were used everywhere except near the region of the singularity. Near the singularity, the function X was approximated by the form

$$\frac{X}{\sqrt{\tau - \tau^*}} = C_1 (\tau - \tau^*) + C_2 (\tau - \tau^*)^2 + C_3 (\tau - \tau^*)^3 + C_4 (\tau - \tau^*)^4 + C_5 (\tau - \tau^*)^5 \quad (A21)$$

where C is a constant.

The total enthalpy was computed from the total gas temperature by the relation

$$\frac{H}{R\theta} = \frac{7}{2} \left(\frac{T}{\theta} \right) + \frac{1}{e^{\theta/T} - 1} \quad (A22)$$

For an isentropic flow

$$\Delta s = \int_{\tau^*}^{\tau} \frac{C_p}{t} dt - R \log_e \frac{p}{p^*} = \Delta S^0 - R \log_e \frac{p}{p^*} = 0 \quad (A23)$$

where

$$\frac{\Delta S^0}{R} = \frac{\tau}{e^{\tau} - 1} - \frac{\tau^*}{e^{\tau^*} - 1} - \log_e \left(\frac{e^{\tau} - 1}{e^{\tau^*} - 1} \right) + \frac{7}{2} \log_e \frac{\tau^*}{\tau} \quad (A24)$$

The ratio of static pressure to initial static pressure at a Mach number of unity was obtained from the relation

$$\frac{p}{p^*} = e^{\frac{\Delta S^0}{R}} \quad (A25)$$

The specific heat at constant pressure was given by

$$\frac{C_p}{R} = \frac{\gamma}{\gamma-1} = \frac{1}{R} \frac{d}{dt} (E_i + Rt) = \frac{7}{2} + \frac{\tau^2 e^\tau}{(e^\tau - 1)^2} \quad (A26)$$

After the pressure, the temperature, and the specific heat, are obtained other variables pertinent to the flow can be determined from fundamental thermodynamic relations.

REFERENCES

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3. Bickley, W. G.: Formulae for Numerical Integration. The Mathematical Gazette, vol. XXIII, no. 256, Oct. 1939, pp. 352-359.

TABLE I - CORNER-FLOW PARAMETERS WITH VARIABLE
SPECIFIC HEAT(a) $T/\theta = 0.3$.

| α (deg) | ϕ (deg) | β (deg) | p/P | M | V/V* | t/T | $\rho V/\rho^* V^*$ |
|-------------------|-----------------|------------------|-------|-------|-------|-------|---------------------|
| 0 | 0 | 90.00 | 0.540 | 1.000 | 1.000 | 0.851 | 1.000 |
| 1 | 23.21 | 67.79 | .491 | 1.080 | 1.067 | .830 | .995 |
| 2 | 29.73 | 62.27 | .462 | 1.130 | 1.108 | .817 | .987 |
| 3 | 34.50 | 58.50 | .437 | 1.173 | 1.142 | .805 | .978 |
| 4 | 38.44 | 55.56 | .415 | 1.213 | 1.174 | .794 | .966 |
| 5 | 41.88 | 53.12 | .394 | 1.250 | 1.202 | .783 | .954 |
| 6 | 44.98 | 51.02 | .376 | 1.286 | 1.230 | .773 | .941 |
| 7 | 47.84 | 49.16 | .358 | 1.322 | 1.256 | .763 | .927 |
| 8 | 50.48 | 47.52 | .341 | 1.356 | 1.280 | .754 | .913 |
| 9 | 52.97 | 46.03 | .325 | 1.390 | 1.304 | .744 | .898 |
| 10 | 55.35 | 44.65 | .310 | 1.423 | 1.328 | .734 | .882 |
| 11 | 57.61 | 43.39 | .296 | 1.456 | 1.350 | .725 | .867 |
| 12 | 59.79 | 42.21 | .282 | 1.488 | 1.372 | .716 | .851 |
| 13 | 61.90 | 41.10 | .269 | 1.521 | 1.394 | .707 | .834 |
| 14 | 63.93 | 40.07 | .256 | 1.554 | 1.415 | .698 | .817 |
| 15 | 65.91 | 39.09 | .244 | 1.586 | 1.435 | .688 | .800 |
| 16 | 67.84 | 38.16 | .232 | 1.618 | 1.455 | .679 | .783 |
| 17 | 69.72 | 37.28 | .221 | 1.651 | 1.475 | .670 | .766 |
| 18 | 71.56 | 36.44 | .210 | 1.684 | 1.495 | .661 | .748 |
| 19 | 73.36 | 35.64 | .200 | 1.716 | 1.514 | .652 | .730 |
| 20 | 75.13 | 34.87 | .190 | 1.749 | 1.533 | .643 | .712 |
| 21 | 76.86 | 34.14 | .180 | 1.782 | 1.551 | .634 | .694 |
| 22 | 78.57 | 33.43 | .171 | 1.815 | 1.569 | .625 | .677 |
| 23 | 80.25 | 32.75 | .162 | 1.849 | 1.587 | .616 | .659 |
| 24 | 81.91 | 32.09 | .154 | 1.882 | 1.605 | .607 | .641 |
| 25 | 83.55 | 31.45 | .146 | 1.916 | 1.622 | .598 | .623 |
| 26 | 85.16 | 30.84 | .138 | 1.951 | 1.640 | .589 | .605 |
| 27 | 86.76 | 30.24 | .131 | 1.985 | 1.656 | .580 | .588 |
| 28 | 88.33 | 29.67 | .124 | 2.020 | 1.673 | .571 | .570 |
| 29 | 89.89 | 29.11 | .117 | 2.056 | 1.690 | .562 | .553 |
| 30 | 91.44 | 28.56 | .110 | 2.092 | 1.706 | .553 | .536 |
| 31 | 92.97 | 28.03 | .104 | 2.128 | 1.722 | .544 | .518 |
| 32 | 94.48 | 27.52 | .098 | 2.164 | 1.738 | .536 | .501 |
| 33 | 95.99 | 27.01 | .092 | 2.202 | 1.754 | .527 | .485 |
| 34 | 97.48 | 26.52 | .087 | 2.239 | 1.769 | .518 | .468 |
| 35 | 98.96 | 26.04 | .082 | 2.278 | 1.785 | .509 | .452 |
| 36 | 100.42 | 25.58 | .077 | 2.316 | 1.800 | .500 | .436 |
| 37 | 101.88 | 25.12 | .072 | 2.356 | 1.815 | .492 | .420 |
| 38 | 103.33 | 24.67 | .068 | 2.396 | 1.829 | .483 | .404 |
| 39 | 104.77 | 24.23 | .064 | 2.436 | 1.844 | .474 | .390 |
| 40 | 106.19 | 23.81 | .060 | 2.478 | 1.858 | .466 | .375 |
| 41 | 107.62 | 23.38 | .056 | 2.520 | 1.873 | .457 | .360 |
| 42 | 109.03 | 22.97 | .052 | 2.562 | 1.887 | .448 | .345 |
| 43 | 110.43 | 22.57 | .049 | 2.606 | 1.900 | .440 | .332 |
| 44 | 111.83 | 22.17 | .045 | 2.650 | 1.914 | .431 | .317 |
| 45 | 113.22 | 21.78 | .042 | 2.695 | 1.928 | .423 | .304 |
| 46 | 114.60 | 21.40 | .039 | 2.741 | 1.941 | .414 | .291 |
| 47 | 115.98 | 21.02 | .037 | 2.787 | 1.954 | .406 | .278 |
| 48 | 117.35 | 20.65 | .034 | 2.835 | 1.967 | .397 | .266 |
| 49 | 118.71 | 20.29 | .032 | 2.884 | 1.980 | .389 | .253 |
| 50 | 120.07 | 19.93 | .029 | 2.933 | 1.993 | .381 | .242 |

TABLE I - CORNER-FLOW PARAMETERS WITH VARIABLE
SPECIFIC HEAT - Concluded(b) $T/\theta = 0.6$.

| α (deg) | ϕ (deg) | β (deg) | p/P | M | V/V* | t/T | $\rho V/\rho^* V^*$ |
|-------------------|-----------------|------------------|-------|-------|-------|-------|---------------------|
| 0 | 0 | 90.00 | 0.546 | 1.000 | 1.000 | 0.867 | 1.000 |
| 1 | 23.09 | 67.91 | .498 | 1.079 | 1.068 | .848 | .995 |
| 2 | 29.57 | 62.43 | .469 | 1.128 | 1.109 | .836 | .987 |
| 3 | 34.32 | 58.68 | .445 | 1.171 | 1.144 | .826 | .978 |
| 4 | 38.24 | 55.76 | .422 | 1.210 | 1.176 | .816 | .966 |
| 5 | 41.68 | 53.32 | .402 | 1.248 | 1.205 | .806 | .954 |
| 6 | 45.03 | 50.97 | .384 | 1.283 | 1.232 | .798 | .941 |
| 7 | 47.65 | 49.35 | .367 | 1.316 | 1.258 | .789 | .928 |
| 8 | 50.47 | 47.53 | .350 | 1.350 | 1.284 | .780 | .913 |
| 9 | 53.03 | 45.97 | .334 | 1.384 | 1.308 | .772 | .898 |
| 10 | 55.26 | 44.74 | .319 | 1.416 | 1.331 | .763 | .883 |
| 11 | 57.29 | 43.71 | .305 | 1.447 | 1.353 | .755 | .868 |
| 12 | 59.50 | 42.50 | .291 | 1.479 | 1.376 | .747 | .852 |
| 13 | 61.60 | 41.40 | .278 | 1.510 | 1.398 | .739 | .835 |
| 14 | 63.59 | 40.41 | .266 | 1.541 | 1.419 | .730 | .819 |
| 15 | 65.51 | 39.49 | .254 | 1.572 | 1.440 | .722 | .802 |
| 16 | 67.42 | 38.58 | .242 | 1.604 | 1.460 | .714 | .785 |
| 17 | 69.29 | 37.71 | .231 | 1.634 | 1.480 | .706 | .768 |
| 18 | 71.11 | 36.89 | .220 | 1.666 | 1.500 | .698 | .750 |
| 19 | 72.89 | 36.11 | .210 | 1.697 | 1.520 | .690 | .733 |
| 20 | 74.64 | 35.36 | .200 | 1.728 | 1.539 | .682 | .715 |
| 21 | 76.35 | 34.65 | .190 | 1.759 | 1.558 | .674 | .698 |
| 22 | 78.04 | 33.96 | .181 | 1.790 | 1.576 | .666 | .680 |
| 23 | 79.71 | 33.29 | .172 | 1.822 | 1.595 | .658 | .662 |
| 24 | 81.35 | 32.65 | .164 | 1.853 | 1.613 | .650 | .645 |
| 25 | 82.96 | 32.04 | .156 | 1.885 | 1.631 | .641 | .628 |
| 26 | 84.56 | 31.44 | .148 | 1.917 | 1.649 | .633 | .610 |
| 27 | 86.14 | 30.86 | .140 | 1.949 | 1.666 | .625 | .592 |
| 28 | 87.70 | 30.30 | .133 | 1.982 | 1.683 | .617 | .575 |
| 29 | 89.24 | 29.76 | .126 | 2.014 | 1.700 | .609 | .558 |
| 30 | 90.76 | 29.24 | .119 | 2.047 | 1.717 | .601 | .542 |
| 31 | 92.27 | 28.73 | .113 | 2.081 | 1.734 | .593 | .524 |
| 32 | 93.77 | 28.23 | .107 | 2.114 | 1.751 | .585 | .508 |
| 33 | 95.25 | 27.75 | .101 | 2.148 | 1.767 | .577 | .492 |
| 34 | 96.72 | 27.28 | .096 | 2.182 | 1.783 | .569 | .475 |
| 35 | 98.18 | 26.82 | .090 | 2.217 | 1.799 | .561 | .459 |
| 36 | 99.63 | 26.37 | .085 | 2.252 | 1.815 | .552 | .444 |
| 37 | 101.07 | 25.93 | .080 | 2.287 | 1.830 | .544 | .428 |
| 38 | 102.50 | 25.50 | .076 | 2.322 | 1.846 | .536 | .413 |
| 39 | 103.91 | 25.09 | .071 | 2.359 | 1.861 | .528 | .398 |
| 40 | 105.32 | 24.68 | .067 | 2.395 | 1.876 | .520 | .383 |
| 41 | 106.72 | 24.28 | .063 | 2.432 | 1.891 | .512 | .369 |
| 42 | 108.11 | 23.89 | .059 | 2.470 | 1.906 | .504 | .355 |
| 43 | 109.50 | 23.50 | .055 | 2.508 | 1.921 | .496 | .341 |
| 44 | 110.88 | 23.12 | .052 | 2.546 | 1.935 | .488 | .328 |
| 45 | 112.25 | 22.75 | .049 | 2.586 | 1.950 | .480 | .314 |
| 46 | 113.61 | 22.39 | .046 | 2.625 | 1.964 | .472 | .301 |
| 47 | 114.97 | 22.03 | .043 | 2.666 | 1.979 | .463 | .289 |
| 48 | 116.32 | 21.68 | .040 | 2.707 | 1.992 | .455 | .276 |
| 49 | 117.66 | 21.34 | .037 | 2.748 | 2.005 | .447 | .264 |
| 50 | 119.00 | 21.00 | .035 | 2.791 | 2.019 | .439 | .252 |

TABLE II - CORNER-FLOW PARAMETERS FOR $\gamma = 1.4$

| α (deg) | ϕ (deg) | β (deg) | p/P | M | V/V* | t/T | $\rho V/\rho^* V^*$ |
|-------------------|-----------------|------------------|-------|-------|-------|-------|---------------------|
| 0 | 0 | 90.00 | 0.528 | 1.000 | 1.000 | 0.833 | 1.000 |
| 1 | 23.38 | 67.62 | .479 | 1.081 | 1.066 | .810 | .995 |
| 2 | 30.02 | 61.98 | .450 | 1.133 | 1.107 | .796 | .986 |
| 3 | 34.78 | 58.22 | .425 | 1.176 | 1.140 | .783 | .977 |
| 4 | 38.85 | 55.15 | .402 | 1.218 | 1.172 | .771 | .965 |
| 5 | 42.35 | 52.65 | .382 | 1.258 | 1.201 | .760 | .953 |
| 6 | 45.38 | 50.62 | .364 | 1.294 | 1.227 | .749 | .940 |
| 7 | 48.25 | 48.75 | .346 | 1.330 | 1.252 | .739 | .926 |
| 8 | 50.90 | 47.10 | .330 | 1.365 | 1.276 | .729 | .912 |
| 9 | 53.42 | 45.58 | .314 | 1.400 | 1.300 | .718 | .897 |
| 10 | 55.83 | 44.17 | .299 | 1.435 | 1.323 | .708 | .881 |
| 11 | 58.10 | 42.90 | .285 | 1.469 | 1.345 | .698 | .865 |
| 12 | 60.30 | 41.70 | .271 | 1.503 | 1.367 | .689 | .849 |
| 13 | 62.40 | 40.60 | .258 | 1.537 | 1.387 | .679 | .832 |
| 14 | 64.45 | 39.55 | .246 | 1.571 | 1.408 | .670 | .815 |
| 15 | 66.45 | 38.55 | .234 | 1.605 | 1.428 | .660 | .798 |
| 16 | 68.38 | 37.62 | .222 | 1.638 | 1.448 | .651 | .780 |
| 17 | 70.27 | 36.73 | .211 | 1.672 | 1.467 | .641 | .762 |
| 18 | 72.12 | 35.88 | .201 | 1.706 | 1.486 | .632 | .744 |
| 19 | 73.93 | 35.07 | .190 | 1.741 | 1.505 | .623 | .726 |
| 20 | 75.73 | 34.27 | .181 | 1.775 | 1.523 | .613 | .708 |
| 21 | 77.47 | 33.53 | .172 | 1.810 | 1.541 | .604 | .690 |
| 22 | 79.18 | 32.82 | .162 | 1.845 | 1.559 | .595 | .672 |
| 23 | 80.87 | 32.13 | .154 | 1.880 | 1.576 | .586 | .654 |
| 24 | 82.52 | 31.48 | .146 | 1.915 | 1.593 | .577 | .635 |
| 25 | 84.17 | 30.83 | .138 | 1.950 | 1.610 | .568 | .617 |
| 26 | 85.77 | 30.23 | .131 | 1.986 | 1.627 | .559 | .600 |
| 27 | 87.38 | 29.62 | .123 | 2.024 | 1.644 | .550 | .581 |
| 28 | 88.97 | 29.03 | .116 | 2.060 | 1.660 | .541 | .563 |
| 29 | 90.52 | 28.48 | .110 | 2.097 | 1.676 | .532 | .546 |
| 30 | 92.03 | 27.97 | .104 | 2.132 | 1.690 | .524 | .530 |
| 31 | 93.60 | 27.40 | .098 | 2.173 | 1.707 | .514 | .511 |
| 32 | 95.10 | 26.90 | .092 | 2.210 | 1.722 | .506 | .494 |
| 33 | 96.60 | 26.40 | .087 | 2.248 | 1.737 | .497 | .478 |
| 34 | 98.08 | 25.92 | .081 | 2.288 | 1.752 | .488 | .461 |
| 35 | 99.58 | 25.42 | .076 | 2.329 | 1.767 | .480 | .444 |
| 36 | 101.03 | 24.97 | .072 | 2.369 | 1.781 | .471 | .428 |
| 37 | 102.50 | 24.50 | .067 | 2.411 | 1.796 | .462 | .412 |
| 38 | 103.93 | 24.07 | .063 | 2.452 | 1.810 | .454 | .397 |
| 39 | 105.38 | 23.62 | .059 | 2.495 | 1.824 | .446 | .381 |
| 40 | 106.80 | 23.20 | .055 | 2.537 | 1.838 | .437 | .366 |
| 41 | 108.20 | 22.80 | .052 | 2.581 | 1.851 | .429 | .352 |
| 42 | 109.62 | 22.38 | .048 | 2.626 | 1.865 | .420 | .337 |
| 43 | 111.02 | 21.98 | .045 | 2.671 | 1.878 | .412 | .323 |
| 44 | 112.40 | 21.60 | .042 | 2.718 | 1.892 | .404 | .309 |
| 45 | 113.78 | 21.22 | .039 | 2.765 | 1.905 | .395 | .295 |
| 46 | 115.17 | 20.83 | .036 | 2.812 | 1.917 | .387 | .282 |
| 47 | 116.53 | 20.47 | .034 | 2.860 | 1.930 | .379 | .270 |
| 48 | 117.92 | 20.08 | .031 | 2.911 | 1.943 | .371 | .257 |
| 49 | 119.27 | 19.73 | .029 | 2.961 | 1.955 | .363 | .245 |
| 50 | 120.62 | 19.38 | .027 | 3.012 | 1.967 | .355 | .234 |

1284

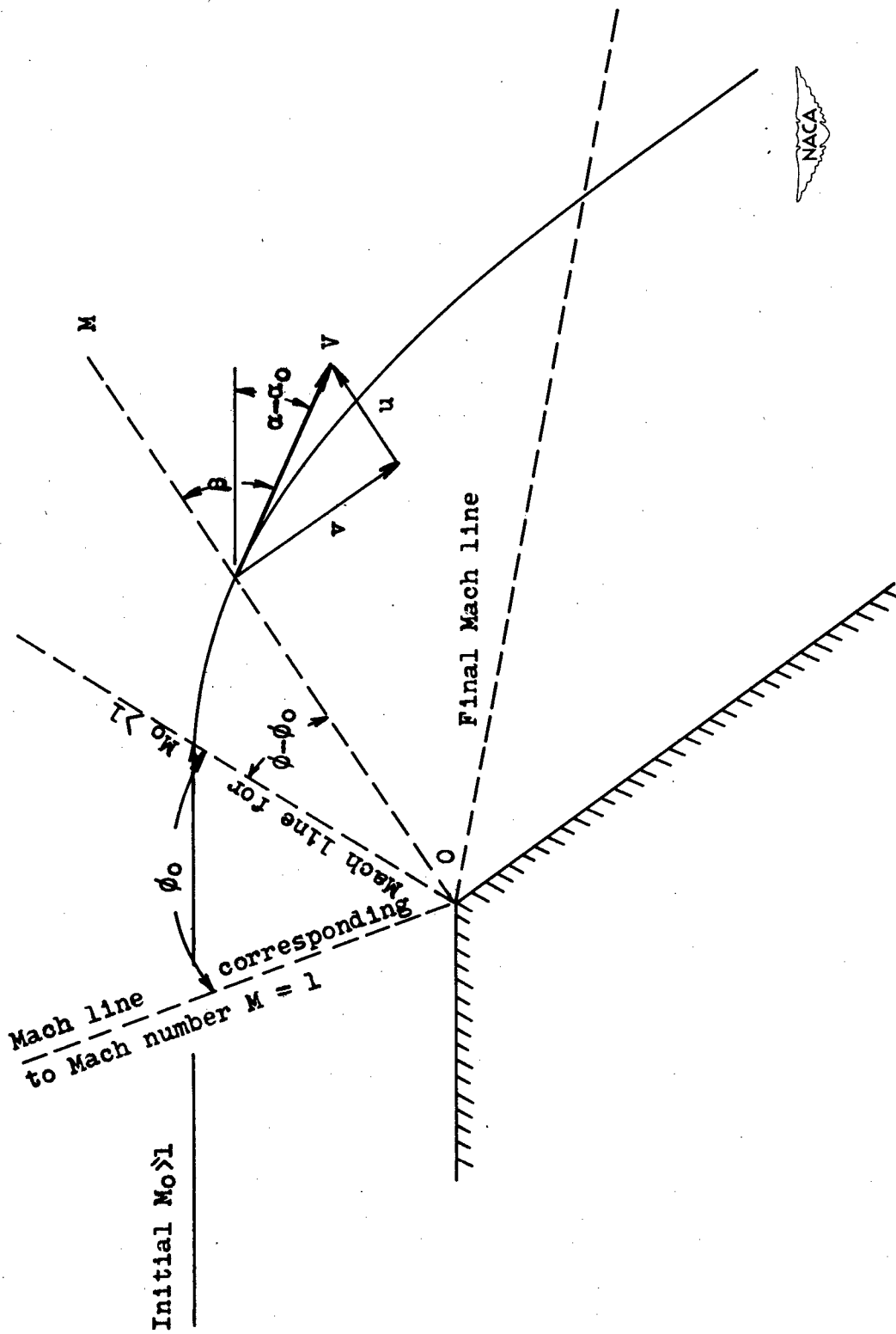


Figure 1. - Schematic representation of supersonic flow around a corner.

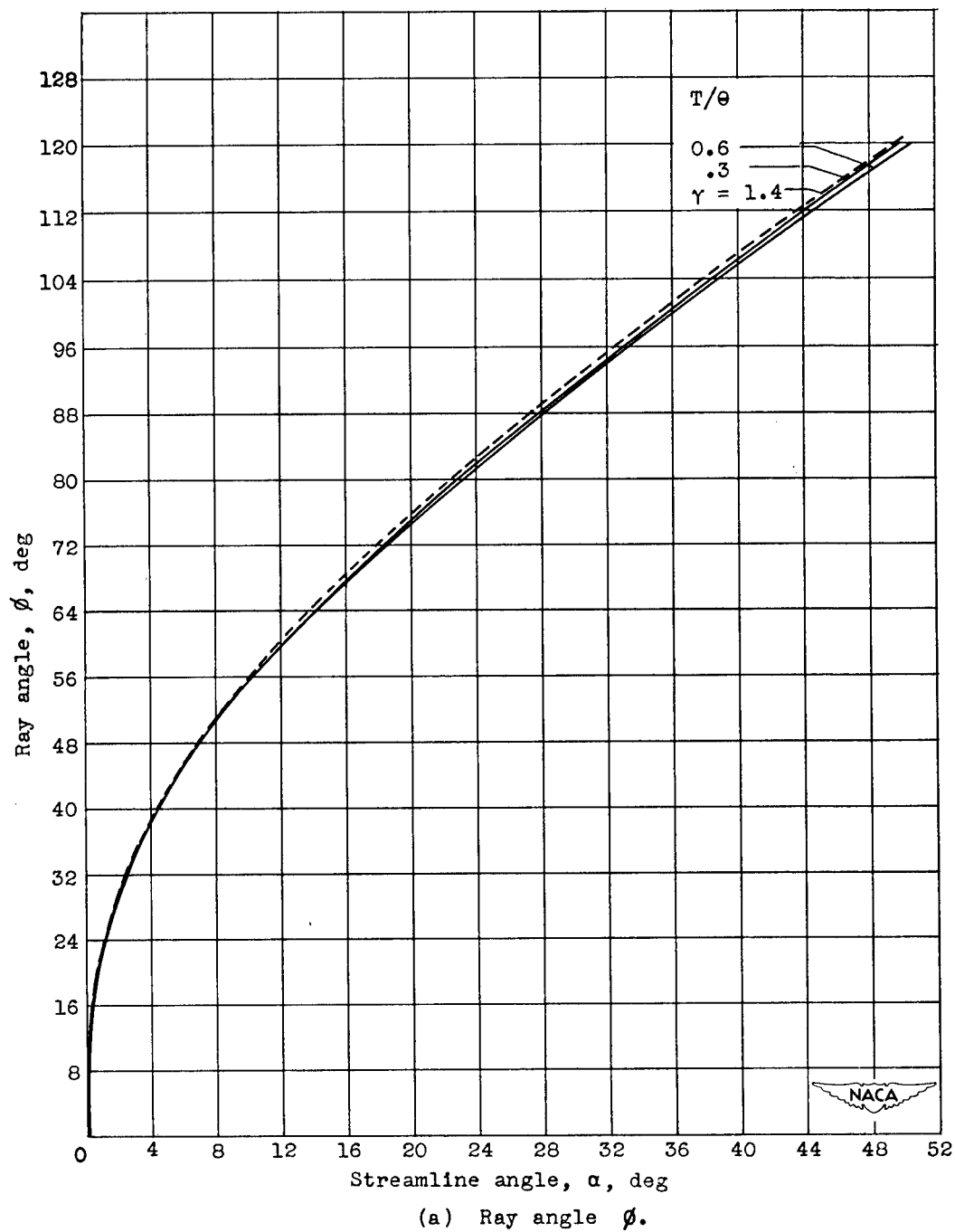


Figure 2. - Variation of corner-flow parameters with streamline angle.
(Data for curve of $\gamma = 1.4$ is taken from reference 2.)

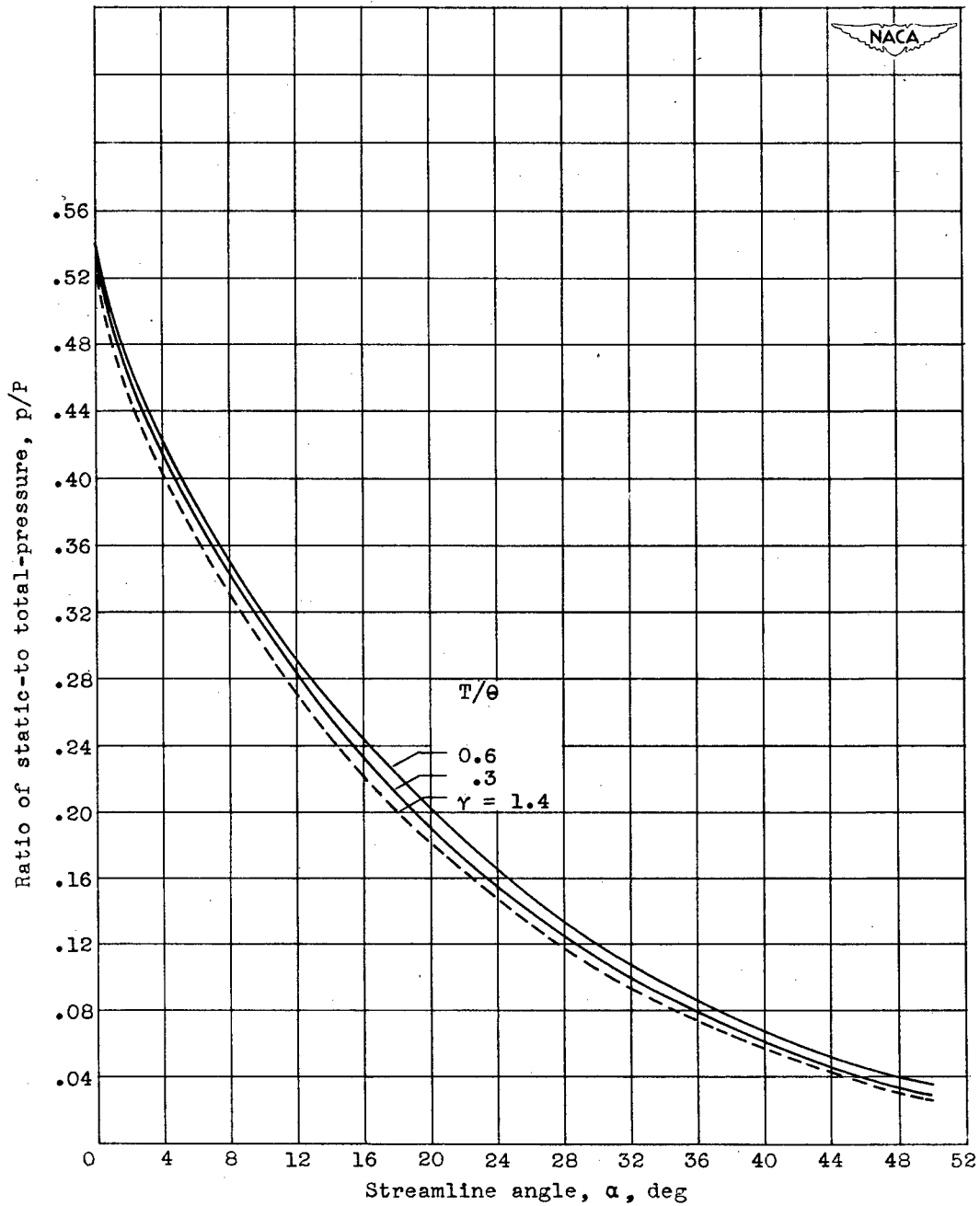
(b) Ratio of static to total pressure p/P .

Figure 2. - Continued. Variation of corner-flow parameters with streamline angle. (Data for curve of $\gamma = 1.4$ is taken from reference 2.)

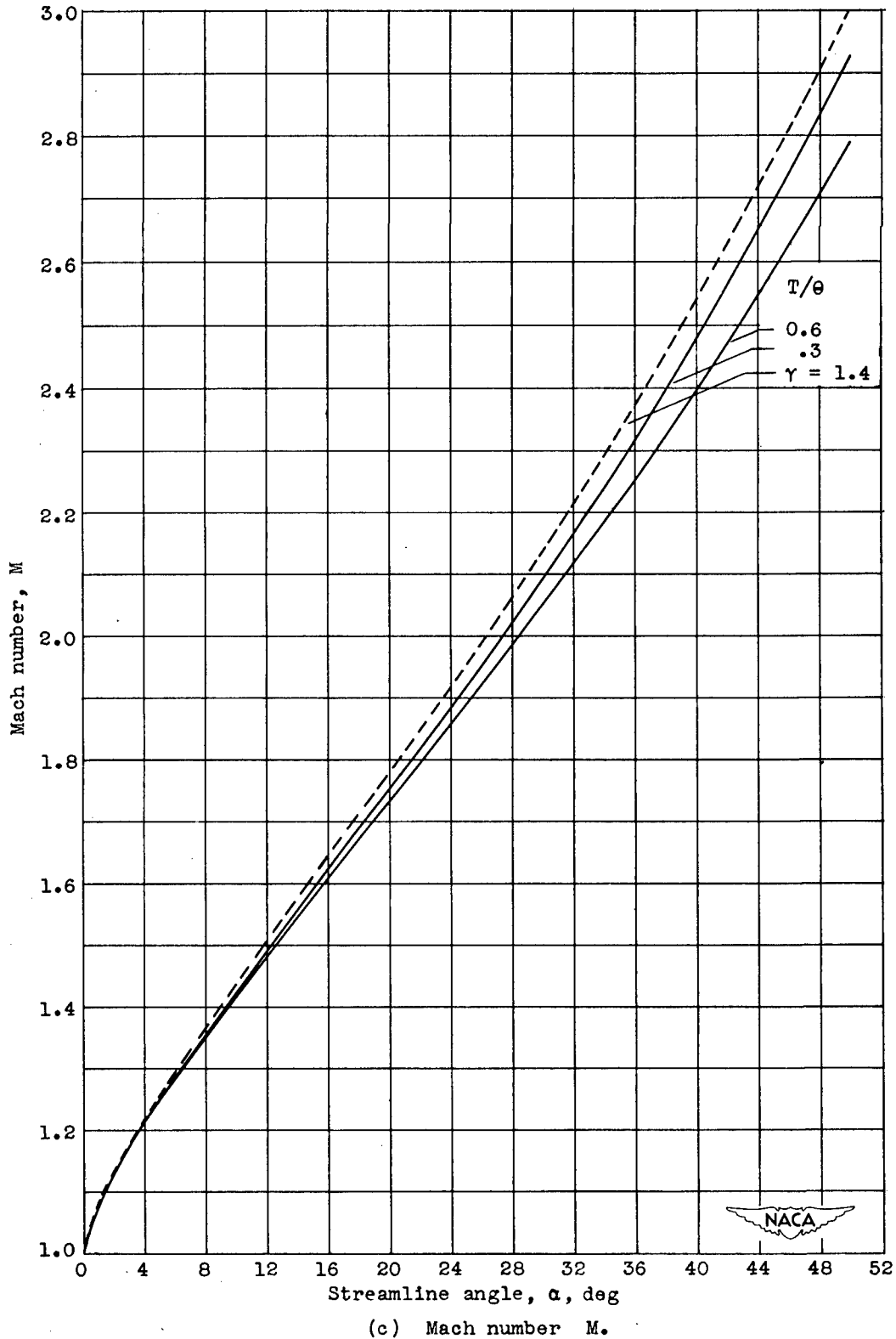
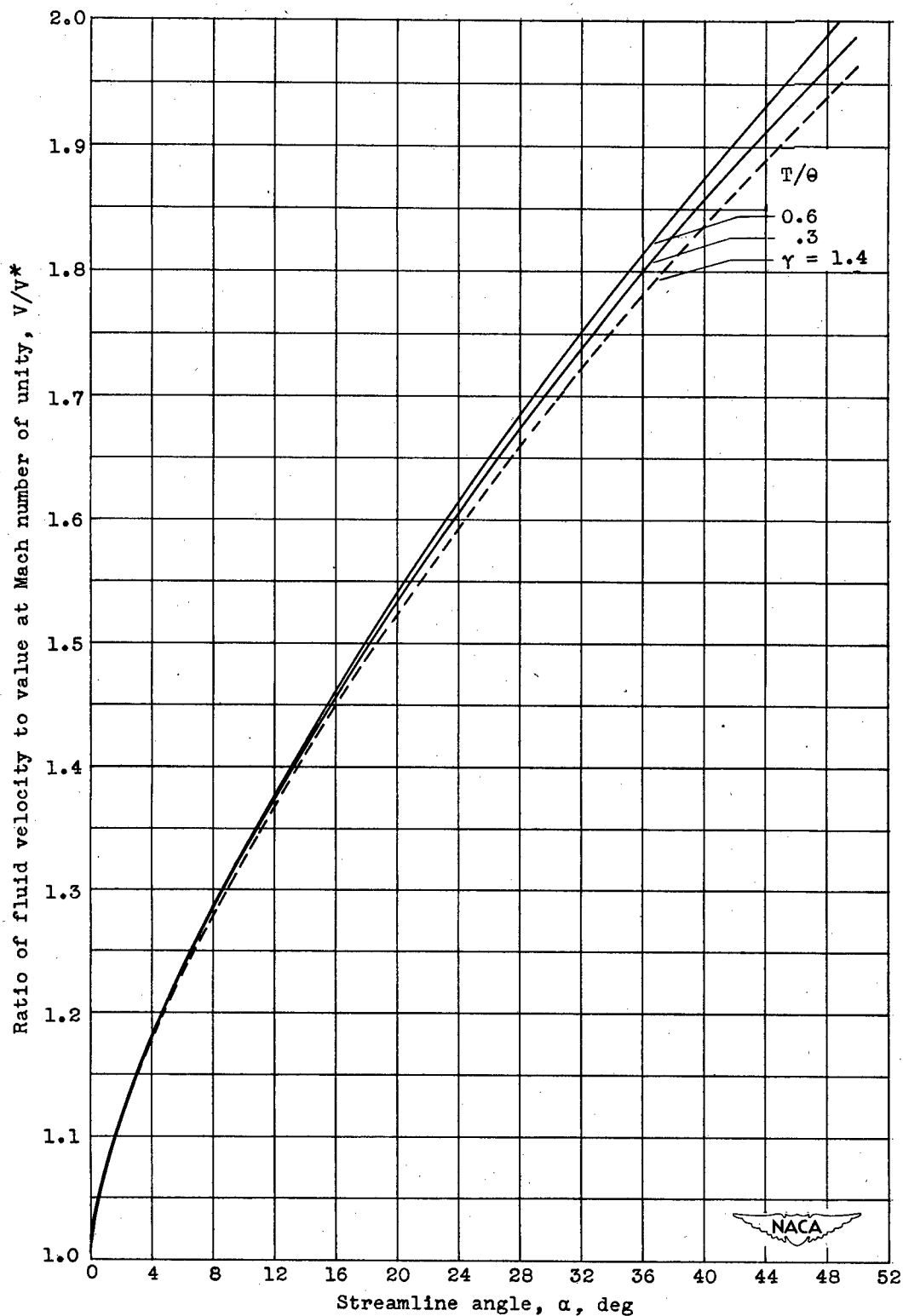
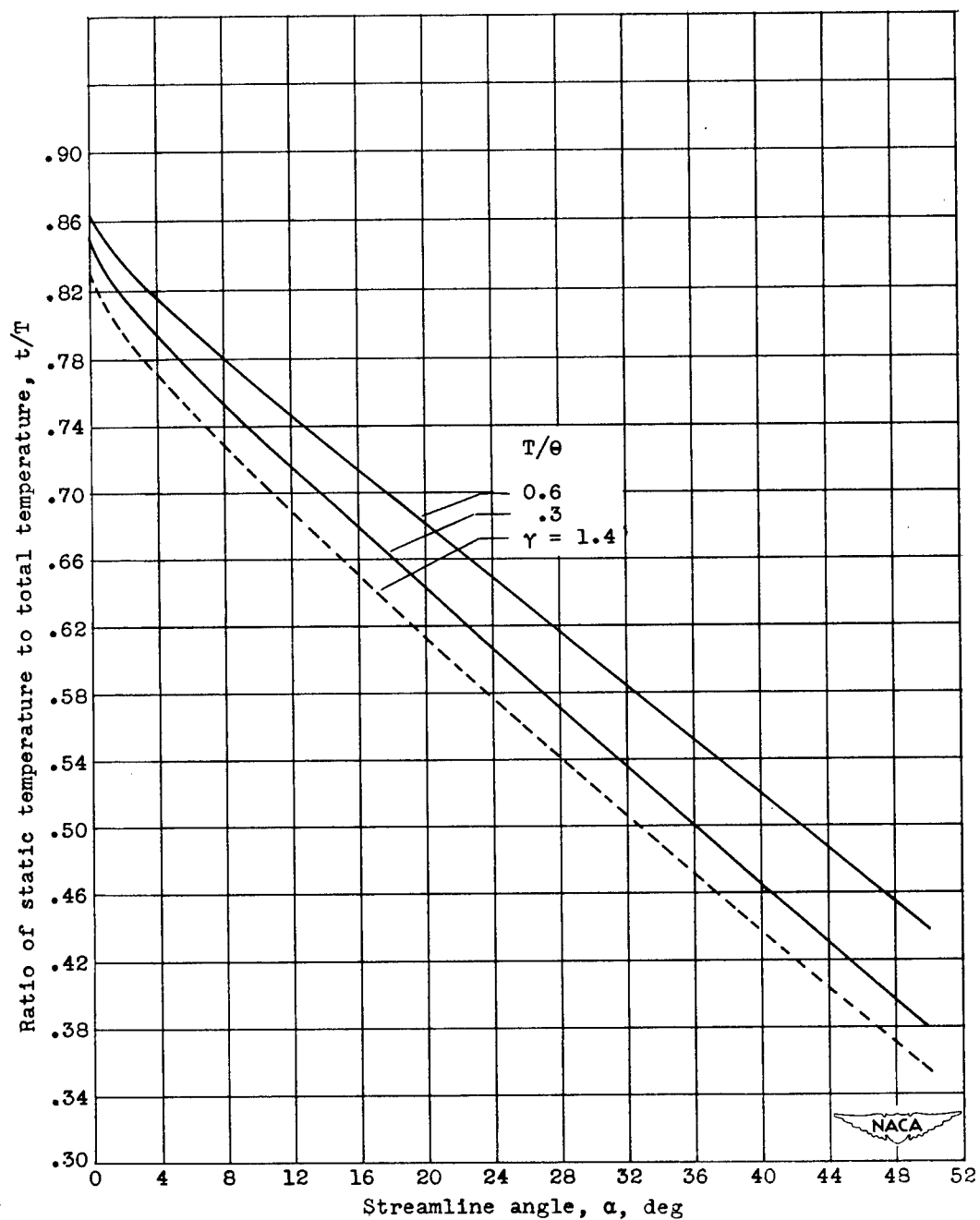


Figure 2. - Continued. Variation of corner-flow parameters with streamline angle. (Data for curve of $\gamma = 1.4$ is taken from reference 2.)



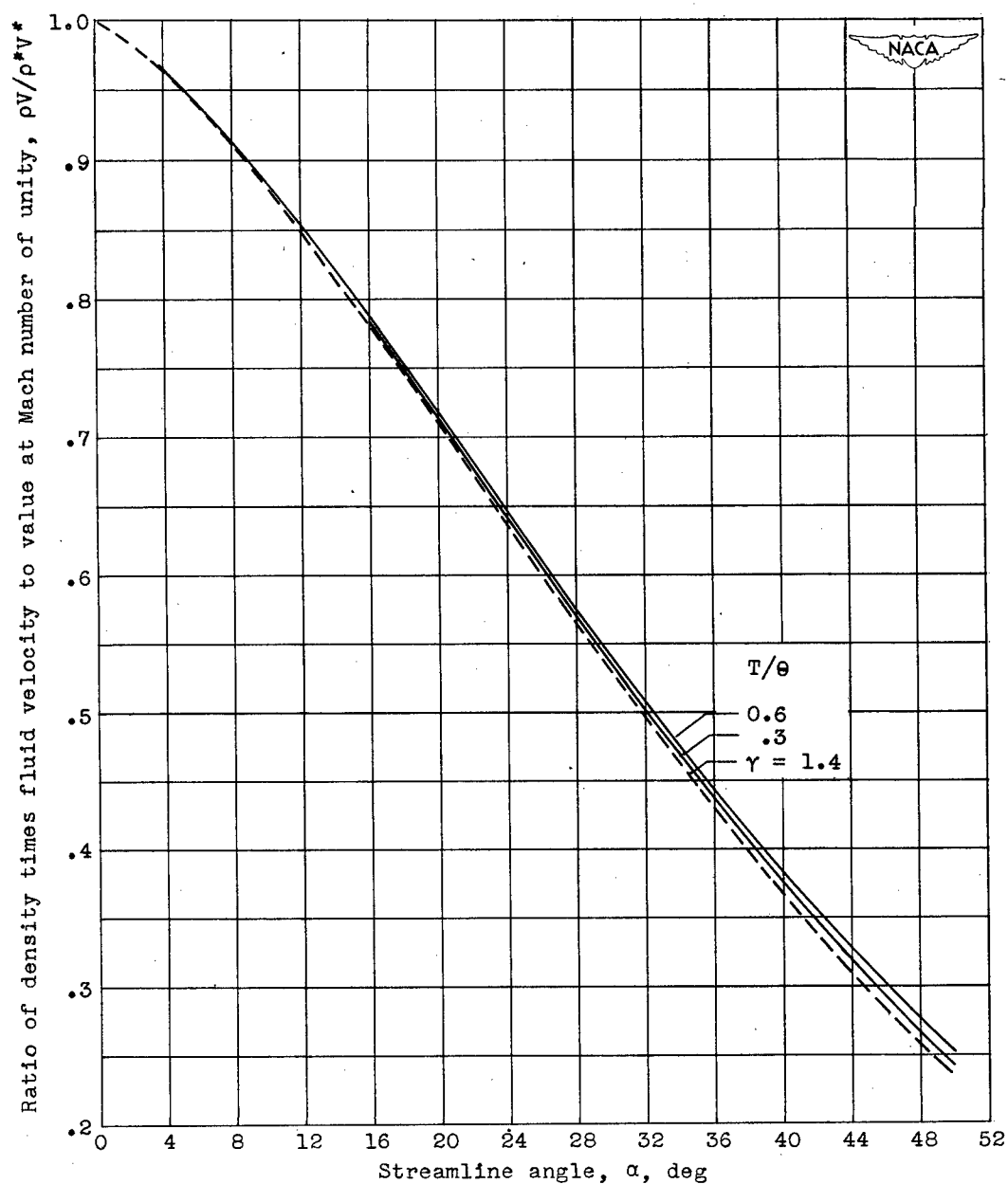
(d) Ratio of fluid velocity to initial value at Mach number of unity V/V^* .

Figure 2. - Continued. Variation of corner-flow parameters with streamline angle. (Data for curve of $\gamma = 1.4$ is taken from reference 2.)



(e) Ratio of static temperature to total temperature t/T .

Figure 2. - Continued. Variation of corner-flow parameters with streamline angle. (Data for curve of $\gamma = 1.4$ is taken from reference 2.)



(f) Ratio of density times fluid velocity to value at Mach number of unity $\rho V / \rho^* V^*$.

Figure 2. - Concluded. Variation of corner-flow parameters with streamline angle. (Data for curve of $\gamma = 1.4$ is taken from reference 2.)

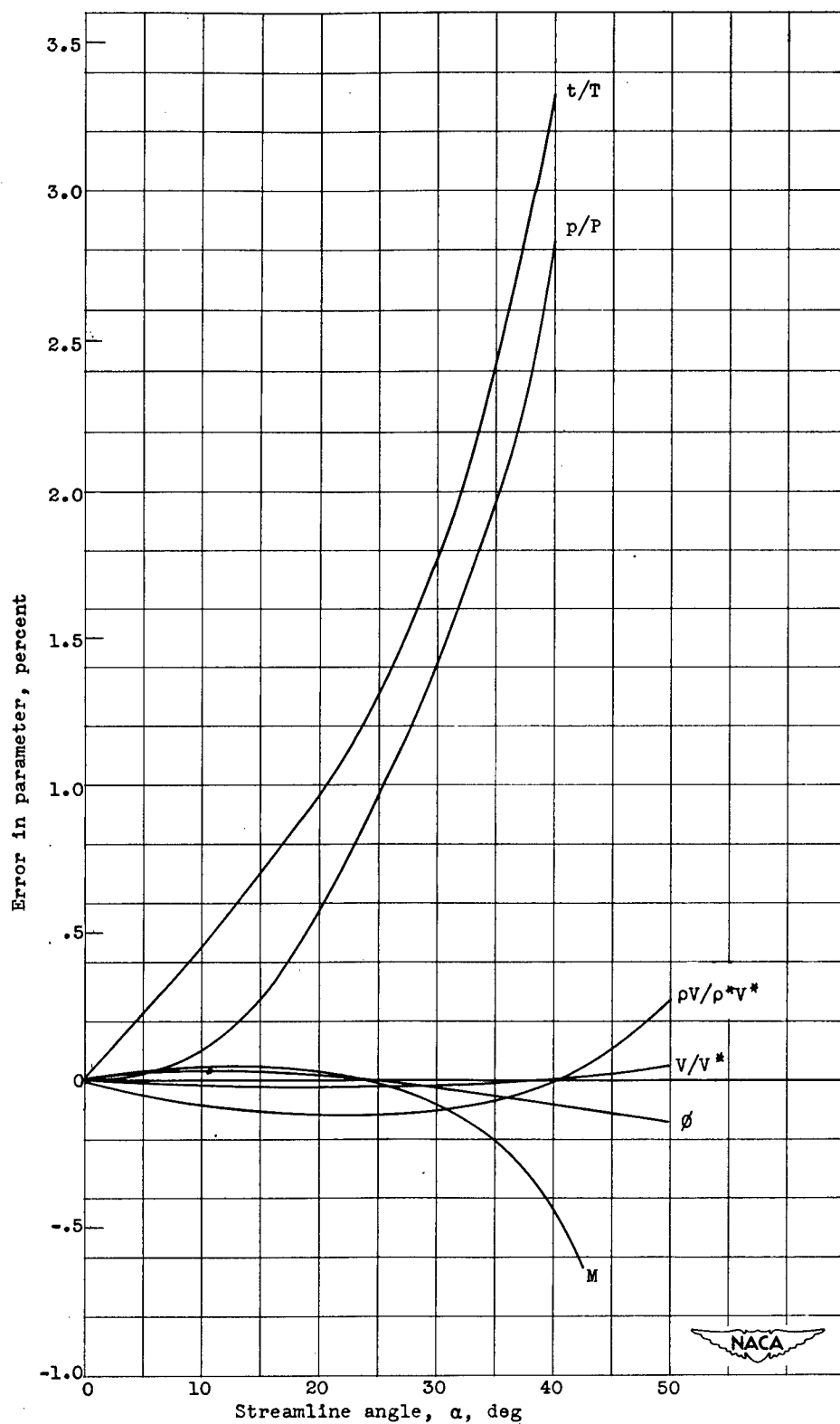


Figure 3. - Error in flow parameters incurred through use of fixed ratio of specific heats ($\gamma = 1.303$) evaluated at total temperature of fluid ($T/\theta = 0.6$).